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The Biasing Effects of Unmodeled ARMA Time Series Processes on Latent Growth Curve Model Estimates

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The purpose of this study was to evaluate the robustness of estimated growth curve models when there is stationary autocorrelation among manifest variable errors. The results suggest that when, in practice, growth curve models are fitted to longitudinal data, alternative rival hypotheses to consider would include growth models that also specify autoregressive (AR), moving average (MA), and autoregressive moving average (ARMA) processes. AR (i.e., simplex) processes are commonly found in longitudinal data and may diminish the ability of a researcher to detect growth if not explicitly modeled. MA and ARMA processes do not affect the fit of growth models, but do notably bias some of the parameters.

This study is intended to identify how robust estimated growth curve model parameters are in the face of stationary autocorrelation among manifest variable errors. Growth curve (GC) models assume temporal manifest variable errors are uncorrelated. This assumption may be tenable on various substantive grounds, although analysts have long known that nuisance correlations among the manifest errors often emerge in longitudinal panel data (e.g., Jöreskog, 1979, 1981; Jöreskog & Sörbom, 1977, 1989; Marsh, 1993; Rogosa, 1979; Sivo, 1997; Sivo & Willson, 1998, 2000). The problem that correlated errors present is that when they

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are not specified the correlated errors systematically bias estimates for the model estimated (Marsh & Grayson, 1993, 1994a, 1994b; Sivo, Pan, & Brophy, 2004). Thus, specification of manifest error correlations is more than simply warranted; it is mandatory.

Sivo (2001) and Sivo and Willson (2000) identified three stationary time series panel models that may be specified for longitudinal data. Although a few academic demonstrations of how to integrate GC models and the autoregressive (AR) model have been discussed in the literature (e.g., Bollen & Curran, 2004; Curran & Bollen, 2001; Rovine & Molenaar, 1998a), the integration of GC models with moving average (MA) and autoregressive moving average (ARMA) panel models as defined in Sivo (2001) and Sivo and Willson (2000) has yet to be presented. This integration is important because the simulation results of Sivo and Willson (2000) suggest that an AR model will not sufficiently account for the effects of autocorrelation due to MA or ARMA. More broadly, the case for regularly evaluating panel data for correlated manifest errors, if it has been made at all, has not been made in a prominent way for analysts employing growth curve modeling. Moreover, even when AR processes are discussed within the context of the GC model, omitted is any discussion of either MA or ARMA processes. Indeed, no systematic study has been conducted to investigate the biasing effects of all three stationary time series processes on GC parameter estimates, and how researchers may model the effects of these processes to reduce their biasing effects. This study accomplishes this, exploiting one of the key features of structural equation modeling (SEM), the allowance not only of measurement error specification, but of their correlations as well.

THEORETICAL FRAMEWORK

The Typical Latent Growth Curve Model

Modeling growth within the SEM framework is a relatively recent approach for studying developmental trends. Because SEM latent growth modeling offers more flexibility in testing different research hypotheses about the developmental trend than some other analytic techniques (e.g., repeated measures analysis of variance), many researchers have argued for its superiority (e.g., Curran, 2000; Duncan, Duncan, Strycker, Li, & Alpert, 1999; Fan, 2003; McArdle & Bell, 2000).

Assuming a series of repeated measurements X_{ti} (t represents the time-ordered measurements of X, and i represents an individual), the latent GC model for describing an individual's growth over this series of repeated measurements is called the *level 1* or *within-person* model:

$$X_{ti} = \alpha_i + \beta_i \lambda_i + \varepsilon_i \tag{1}$$

where α_i is the intercept of an individual's growth trajectory (i.e., the initial status measured at Time 1), β_i is the slope of the individual's growth trajectory (i.e., the unit change in X_i between two consecutive measurements), λ_i represents the consecutive measurement time points, and ε_i represents the modeling residual for an individual.

Because the intercept (α) and the slope (β) are random variables, these individual model parameters can be represented by the group mean intercept (μ_α) and group mean slope (μ_β) plus individual variation ($\zeta_{\alpha i}$, $\zeta_{\beta i}$) in the following *Level 2* or *between-person* model:

$$\alpha_i = \mu_\alpha + \zeta_{\alpha i}$$
$$\beta_i = \mu_\beta + \zeta_{\beta i} \tag{2}$$

The Level 2 model is often called the *unconditional model* (e.g., Curran, 2000), and it assumes that no other predictors in the model account for the variation of individual intercepts and slopes. The unconditional latent GC model (for eight repeated measurements) is represented graphically as the aspect of the SEM model in Figure 1 excepting the dotted lines.

In a latent GC model as shown in Figure 1 (dotted lines excluded), typically, no nonzero covariance structure is hypothesized for the residuals (e_1–e_8) of the observed variables (X_1–X_8). In most applications, the covariance matrix for the residuals (e_1–e_8) is simply assumed to be a diagonal matrix.

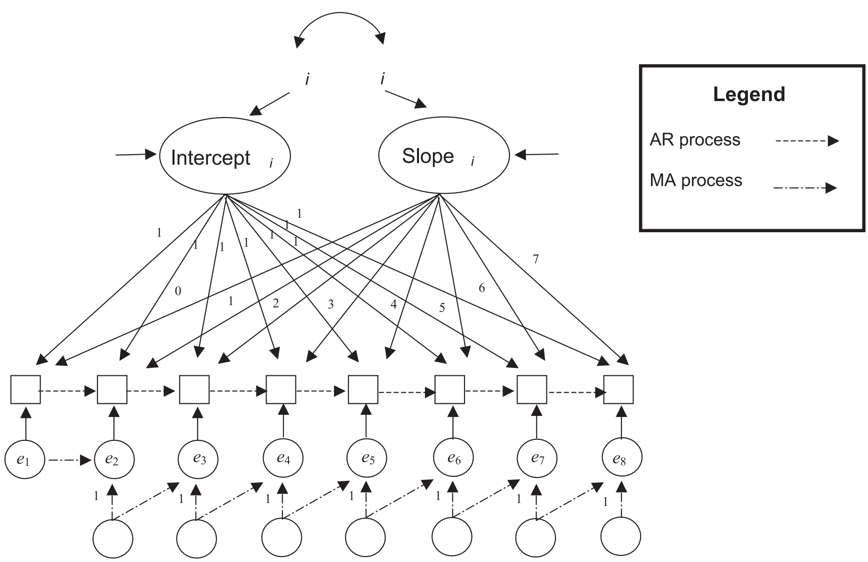

FIGURE 1 The linear growth curve with possible first-order autoregressive moving average processes.

The latent GC model falls within the general class of models addressing nonstationary temporal processes. Each of the five parameters of the unconditional model provides answers to very specific questions from which the applied researcher can greatly benefit. The intercept mean indicates the average starting point for the people under study with respect to the variable in question. The slope mean indicates the average rate of change over time for the people under study with respect to the variable in question. The intercept variance indicates the degree that people vary at the start of the study with respect to the variable in question. The slope variance indicates the degree to which people vary in terms of their rate of change over time with respect to the variable in question. The correlation between the intercept and slope indicates how much of a relation exists between participants' starting points in the study and their rate of change with respect to the variable in question.

Interpretation of these five growth parameters is fairly straightforward so long as the particular covariance structure hypothesized for the residuals (e_1–e_8) is a diagonal matrix. In the context of this study, several questions are raised when residuals are autocorrelated: (a) How confident can we be in our assessment of the growth parameters when autocorrelated errors are present in the growth data? (b) How does one detect the degree of bias caused by autocorrelated errors? (c) What theoretically plausible solutions exist for treating such a nuisance condition?

Stationary Time Series

Stationary time series data may evidence the presence of two processes: AR and MA (Box & Jenkins, 1976). AR models answer the question, "How is the stability of a construct over time affected by autocorrelated observed scores?" AR models are specified to represent the current value of a time series as a function of previous values of the same time series. Generally,

$$X_t = \alpha_i X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} + \varepsilon_t \tag{3}$$

where X_t denotes an observed score taken at some time point (t) following after the original level X_0 of the series, α denotes a correlation among temporally ordered scores at some lag (e.g., $t-1$ = a lag of 1, $t-2$ = a lag of 2), and ε denotes residual associated with a given occasion (t).

MA models answer the question, "How is the stability of a construct over time affected by autocorrelated residuals?" MA models are specified to represent the current value of a time series as a function of autocorrelated residuals. Generally,

$$X_t = \varepsilon_t - \beta_1 \varepsilon_{t-1} - \beta_2 \varepsilon_{t-2} - \ldots - \beta_q \varepsilon_{t-q} \tag{4}$$

where X_t denotes an observed score taken at some time point (t) following after the original level X_0 of the series, β denotes a correlation among residuals at some lag (e.g., $t-1$ = a lag of 1, $t-2$ = a lag of 2), and ε denotes a residual associated with a given occasion (t). In the context of SEM, there has been some evidence that the log likelihood ratio for estimates of pure MA processes is not reliable when such models are fitted to Toeplitz matrices (Hamaker, Dolan, & Molenaar, 2002). However, within the context of panel models designed to estimate MA processes, no such problem has been found (Sivo, 2001; Sivo & Willson, 2000). Not only is the log likelihood ratio for MA panel models reliable, but the estimation of the MA processes is unbiased and efficient.

The possibility of both processes being present in the same data supports the specification of ARMA models:

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} - \beta_1 \varepsilon_{t-1} - \beta_2 \varepsilon_{t-2} - \ldots - \beta_q \varepsilon_{t-q} + \varepsilon_t \quad (5)$$

ARMA models answer the question, "How is the stability of a construct over time affected by autocorrelated observed scores and residuals?" Time series models have been increasingly used in the context of SEM in various contexts. With respect to longitudinal panel data, time series models for stationary processes have been discussed for both single-indicator models (Sivo & Willson, 2000) and multiple-indicator models (Sivo, 2001). Although such models have the potential of being compatible with GC models, indeed, addressing issues pertinent to the analysis of all longitudinal data, a full integration of the two types of models has yet to be discussed. Although there has been some attempt to integrate AR (i.e., simplex) models with growth models (e.g., Bentler, Newcomb, & Zimmerman, 2002; Curran & Bollen, 2001), a full integration has yet to be developed, despite the benefit that an integration of MA and ARMA models offers in the face of correlated temporal errors, the estimate-biasing, nuisance condition all too common to longitudinal panel data. As Sivo (2001) and Sivo and Willson (2000) indicated, MA and ARMA models allow for the asymmetric specification of correlated residuals.

This study accomplishes two purposes. First, this study introduces how GC models and MA and ARMA models may be integrated, information previously not found in the literature. Second, this study investigates the degree to which autocorrelation in its various forms (AR, MA, and ARMA) biases the estimates obtained in GC modeling.

Scientific Importance of the Study

This study focuses on a very important issue that has yet to be considered in the GC modeling literature. What do we do in the face of autocorrelated residuals, so common to longitudinal data, when we desire to model growth over time? Some studies academically consider the integration of AR and growth models, but no studies consider the integration of its MA and ARMA counterparts as specified in Sivo (2001) and Sivo and Willson (2000). MA and ARMA models allow for the asymmetric specification of correlated residuals. No other GC study handles this issue, and previous research testifies to the importance of this issue and the viability of the modeling approach advocated in this article. Given that correlated errors can bias other parameter estimates, and that they are so prevalent in longitudinal data, how often has GC modeling been employed in practice without the consideration of MA and ARMA processes, not to mention AR processes?

Research Questions

This study examines how five GC parameter estimates (intercept mean, slope mean, intercept variance, slope variance, and intercept–slope relation) are affected by autocorrelation. Specifically, the following research questions are answered in this study:

1. Which of the five GC parameter estimates are affected by unmodeled first-order autoregression (AR1)?
2. Which of the five GC parameter estimates are affected by an unmodeled first-order moving average (MA1) process?
3. Which of the five GC parameter estimates are affected by an unmodeled first-order autoregressive moving average (AR1-MA1) process?

METHOD

Data Source

The simulation macro program developed using SAS software for the study has been designed to allow for 200 replications for each of six parametric conditions for a total of $4 \times 4 \times 2 \times 2 \times 2 \times 4 = 512$ research conditions. In all conditions, the intercept mean was 1 and the slope mean was 1. The conditions considered include the following:

1. Four models (GCM, GCAR, GCMA, GCARMA).
2. Sample size (150, 250, 350, 450).
3. Intercept variance (.2, .7).
4. Slope variance (.1, .5).
5. Phi between the intercept and slope (.00, .50).
6. Squared AR/MA coefficients (AR/MA: .00/00; .25/.00; .00/.20; .25/.20).

Clearly, many more gradations of parametric conditions were possible, but the values were chosen to be large enough to allow for an identification of notable findings. This selection of values is believed to be discrepant enough to answer the initial inquiry of whether differences exist, while protecting the study from becoming unmanageable for interpretation. Follow-up analyses were conducted to explore further the findings when notable differences were found.

It is important to point out that Rovine and Molenaar (1998b), as well as Stoel and van den Wittenboer (2003), found that the covariance between the intercept and slope (phi) is affected by the basis parameters for the time scale chosen when fitting a model to the data. So, correct estimation of the phi in practice depends on the basis parameters. The results of the simulations conducted in this study are based on the assumption the researcher is using the correct basis parameters. So, conclusions about the effect of either the AR or MA process on the estimation of phi are accurate assuming the correct basis parameters are chosen.

The data generated for this study consisted of eight occasions. Eight wave data sets were generated for a number of reasons. The conditions for a Monte Carlo study must be delimited carefully or else the summary of the findings can be unwieldy. Consequently, Monte Carlo studies are better designed when they answer a few critical questions well. The purpose of this study is to determine the effect of unmodeled ARMA processes on GC parameter estimates. A generous number of occasions were considered so that model comparisons could be more likely when differences truly exist. Should unmodeled ARMA processes be determined to affect GC parameter estimates, a later study will be implemented to examine the limits of such findings.

Procedure

In this study, four latent GC models are estimated for simulated data. The form of all four models may be visualized in Figure 1, where AR and MA process are presented with different styled dotted lines (see the figure legend). The four models include the growth curve model (GC), the growth curve autoregressive (GCAR) model, the growth curve moving average (GCMA) model, and the growth curve autoregressive moving average (GCARMA) model. All four eight-wave models are estimated using the SAS Institute's (1989) PROC CALIS (Covariance Analysis of Linear Structural Equations).

The research questions are answered by examining the results of a Monte Carlo simulation study. Data for each combination of parametric conditions were generated using SAS Institute's (1989) RANNOR function. The procedure for generating the AR and MA processes is described in the SAS book, *SAS for Monte Carlo Studies: A guide for quantitative researchers* (Fan, Felsovalyi, Sivo, & Keenan, 2002). It is worth noting that an alternative approach to answering the research questions would have been to record the differences found between the results of the rival models fitted to the population matrices. In practice, however, random sampling fluctuations can make it difficult for one to distinguish models that belong to the same family of models. Box and Jenkins (1976) made this point specifically about AR and MA processes. Demonstrating that the models may be differentiated under random conditions builds the case for the discriminant validity of the models in practice. For these results to be most practicable, it is important to examine what difference can be found in the results of the rival models when conditions are imperfect. One of the purposes of a Monte Carlo study is to examine what happens when statistical assumptions are systematically violated (Fan et al., 2002). This study is intended to examine how the systematic violation of the assumption of no autocorrelation affects, on average, the estimated GC parameters. It is important to note whether the differences in the rival model results for the GC parameters are of such a magnitude that they are evident even in the face of random sampling fluctuations.

In addition to inspecting the accuracy of the parameter estimates, the following fit indexes were examined: the Goodness of Fit Index (GFI), Comparative Fit Index (CFI), McDonald's Centrality Index (Mc), and the Standardized Root Mean Square Residual Estimate (SRMR). These indexes were chosen because of their relative merits. The GFI and Mc are stand-alone indexes that have a long history in SEM research. The CFI is an incremental fit index that indicates how much the fit of a model improves on the nested null model. The SRMR summarizes the residual variation.

RESULTS AND DISCUSSION

The results are organized so that the effects of each type of autocorrelation are described in the following order: AR, MA, and ARMA. The research questions for the study were answered by comparing the results obtained for the GC model when fitted to the AR, MA, or ARMA data to the proper model for each data set. In each case, the biasing effects of the autocorrelation are first described, followed by a comparison of the fit results. Although the intercept and slope means were held constant, each at a value of 1, it was informative to examine the degree to which an autocorrelation affected their estimation.

The Effect of Unmodeled Lag One AR on GC Model Parameter Estimates

These results were obtained by comparing the fit of the GC model and the GCAR model to data with both an AR and GC process. When the effects of AR on the GC parameters were examined it was readily apparent that estimates were noticeably biased upward. The mean differences between the parameter estimated for the GC model and the GCAR model (i.e., the correct model for the data) were all statistically significant ($p < .05$; see Table 1). In fact, the between the intercept mean increased on average by 77%, and the slope mean increased by more than 200% (see Table 2). The intercept variance increased by more than 300%, and this was true re-

TABLE 1
GC Model and GCAR Model Differences With Respect to Growth Curve Parameter Estimates

Source	*df*	*Anova SS*	*Mean Square*	*F Value*	*Pr > F*
Dependent variable: Intercept mean					
Model	1	5057.437445	5057.437445	44458.4	<.0001
N	3	0.045132	0.015044	0.13	0.9409
Model * *N*	3	0.011329	0.003776	0.03	0.9919
Error	12,792	1455.174933	0.113757		
Dependent variable: Slope mean					
Model	1	180.6864967	180.6864967	51871.1	<.0001
N	3	0.0019168	0.0006389	0.18	0.9077
Model * *N*	3	0.0009701	0.0003234	0.09	0.9640
Error	12,792	44.5593630	0.0034834		
Dependent variable: Intercept variance					
Model	1	17320.07387	17320.07387	16737.3	<.0001
int_var	1	8248.07190	8248.07190	7970.54	<.0001
Model * int_var	1	369.07834	369.07834	356.66	<.0001
Error	1,2796	13241.54997	1.03482		
Dependent variable: Slope variance					
Model	1	1.75523214	1.75523214	18735.9	<.0001
slp_var	1	0.03392137	0.03392137	362.09	<.0001
Model * slp_var	1	0.02867898	0.02867898	306.13	<.0001
Error	12,796	1.19876369	0.00009368		
Dependent variable: Correlation between the intercepts and slopes					
Model	1	30.99129777	30.99129777	24433.8	<.0001
phi	1	7.24836538	7.24836538	5714.66	<.0001
Model * phi	1	15.79171523	15.79171523	12450.3	<.0001
Error	12,796	16.23018843	0.00126838		

Note. The effects considered in the analysis include model (GCAR vs. GC model), *N* (150, 250, 350, 450), int_var (.2, .7), slp_var (.1, .5), and phi (.00, .50).

gardless of the value of the intercept variance (.2 or .7). AR likewise increased the slope variance by more than 300% regardless of the size of the slope variance (.1, .5). Finally, the average correlation found between the slope and intercept increased on average by more than 200%. Furthermore, when the phi value was set to 0, a very slight correlation between these two parameters was estimated to exist, with an average of about .10.

The fit results for the GC model were notably poor when fit to the GCAR data. The GFI was on average, less than .50. The CFI results for the GC model were less than .81, the Mc less than .70, and the SRMR was greater than .30.

A follow-up analysis in which the AR parameter coefficient value was adjusted downward to .15 and .05 reveals that the GC model begins to fit acceptably (e.g.,

TABLE 2
The Biasing Effect of Autoregression on Growth Parameters

Parameter	*% Parameter Bias Due to Autoregression*
Intercept mean	77
Slope mean	> 200
Slope variance	> 300
Intercept variance	> 300
Correlation found between the slope and intercept	> 200

Note. The data were generated to have a mean intercept and slope of 1; other parameters were varied—intercept variance (.2, .7), slope variance (.1, .5), and the phi between the intercept and slope (.00, .50)

CFI > .95 and SRMR < .05) with small AR lag values, although the GC estimates continue to be biased upward to a noticeable degree.

The Effect of an Unmodeled Lag One MA Process on GC Model Parameter Estimates

These results were obtained by comparing the fit of the GC model and the GCMA model to data with both an MA and GC process. The GC parameter estimates in the face of an MA process were different than those for the AR process. The intercept mean, slope mean, and slope variance were all unbiased. Not only were these growth parameters estimated for the GC model and GCMA each not different to a statistically significant degree ($\alpha = .05$), but the estimates were nearly the same. These findings may be seen in Table 3 when reported model effects are reviewed. Other effects not of primary concern, such as sample size or parameter condition, were modeled as a form of control, and so, although the effects are reported, they are not addressed.

On the other hand, the intercept variance was biased upward on average by .10. The largest impact on the intercept variance was smaller (set at .2 as opposed to .7). The introduction of the MA process to the GC model actually led to a modest underestimation of the correlation between the intercept and slope by .05, on average. This is a result of the increase in the intercept variance in the presence of the MA process. With the intercept variance increasing and the slope variance remaining unaffected, the end result is a smaller estimated correlation between the two parameters.

A follow-up analysis in which the MA parameter coefficient values were set at .10, .30, and .50 revealed that the intercept variance and intercept slope correlation values were very biased even when the MA process was dropped to a coefficient of

TABLE 3
GC Model and GCMA Model Differences With Respect to Growth Curve Parameter Estimates

Source	*df*	*Anova SS*	*Mean Square*	*F Value*	*Pr > F*
Dependent variable: Intercept mean					
Model	1	0.00010735	0.00010735	0.03	0.8668
N	3	0.03808785	0.01269595	3.33	0.0188
Model * *N*	3	0.00028075	0.00009358	0.02	0.9948
Error	12,792	48.82952486	0.00381719		
Dependent variable: Slope mean					
Model	1	0.00000468	0.00000468	0.01	0.9386
N	3	0.00418631	0.00139544	1.77	0.1502
Model * *N*	3	0.00002185	0.00000728	0.01	0.9988
Error	12,792	10.07488076	0.00078759		
Dependent variable: Intercept variance					
Model	1	0.9188323	0.9188323	36.67	<.0001
int_var	1	271.8176642	271.8176642	10848.1	<.0001
Model * int_var	1	0.1031636	0.1031636	4.12	0.0425
Error	12,796	320.6267668	0.0250568		
Dependent variable: Slope variance					
Model	1	0.04712015	0.04712015	3.14	0.0764
slp_var	1	0.47372078	0.47372078	31.56	<.0001
Model * slp_var	1	0.19712427	0.19712427	13.13	0.0003
Error	12,796	192.04215080	0.0150080		
Dependent variable: Correlation between the intercepts and slopes					
Model	1	0.2719933	0.2719933	90.15	<.0001
phi	1	269.4746277	269.4746277	89316.5	<.0001
Model * phi	1	0.7409183	0.7409183	245.58	<.0001
Error	12,796	38.6064867	0.0030171		

Note. The effects considered in the analysis include model (GCMA vs. GC model), *N* (150, 250, 350, 450), int_var (.2, .7), slp_var (.1, .5), and phi (.00, .50).

.10. The intercept variance became more biased upward as the MA process increased, and the intercept slope correlations, in turn, decreased.

Unlike the fit results obtained for the AR process, the GC model when fit to GCMA data received fit results that were high relative to the correct model. Despite the bias in the two parameter estimates, the fit indexes for the GC model to the GCMA data were very good (e.g., CFI > .95, SRMR < .05) when fit to data with an MA process of .20. The CFI and SRMR failed to detect a difference between the models whatsoever. The GFI and Mc both detected a slight difference, with coefficient differences on average ranging from .01 or .02 coefficient points.

The Effect of an Unmodeled Lag One ARMA Process on GC Model Parameter Estimates

These results were obtained by comparing the fit of the GC model and the GCARMA model to data with both an ARMA and GC process. Results obtained when the GC model fit to the GCARMA data were very similar to those obtained for the GCMA data (see Table 4). The intercept mean, slope mean, and slope variance were all unbiased. The intercept variance and the slope–intercept correlation were biased, with the upward bias in the intercept variance accounting for the downward bias of the slope–intercept correlation.

TABLE 4
GC Model and GCARMA Model Differences With Respect to Growth Curve Parameter Estimates

Source	*df*	*Anova SS*	*Mean Square*	*F Value*	*Pr > F*
Dependent variable: Intercept mean					
Model	1	0.76279598	0.76279598	63.15	<.0001
N	3	0.12583324	0.04194441	3.47	0.0154
Model * *N*	3	0.18012238	0.06004079	4.97	0.0019
Error	12,792	154.51292310	0.0120789		
Dependent variable: Slope mean					
Model	1	3703.703336	3703.703336	6747.47	<.0001
N	3	8.367910	2.789303	5.08	0.0016
Model * *N*	3	8.122422	2.707474	4.93	0.0020
Error	12,792	7021.555950	0.54890		
Dependent variable: Intercept variance					
Model	1	151.3096557	151.3096557	2121.84	<.0001
int_var					
Model * int_var	1	3.3125023	3.3125023	46.45	<.0001
Error	12,796	912.4922240	0.071311		
Dependent variable: Slope variance					
Model	1	197.0269376	197.0269376	1505.10	<.0001
slp_var	1	64.2834644	64.2834644	491.06	<.0001
Model * slp_var	1	15.2065968	15.2065968	116.16	<.0001
Error	12,796	1675.0782850	0.130906		
Dependent variable: Correlation between the intercepts and slopes					
Model	1	220.0676988	220.0676988	6254.60	<.0001
phi	1	416.3380048	416.3380048	11832.8	<.0001
Model * phi	1	22.9068449	22.9068449	651.04	<.0001
Error	12,796	450.2264640	0.035185		

Note. The effects considered in the analysis include model (GCARMA vs. GC model), *N* (150, 250, 350, 450), int_var (.2, .7), slp_var (.1, .5), and phi (.00, .50).

A follow-up analysis in which the AR and MA parameter coefficient values were set at .10, .30, and .50 revealed that the intercept variance and the intercept–slope correlation continued to be very biased even when both the AR and MA coefficients were both dropped to a value of .10.

The fit results obtained for the GC model were very competitive with the GCARMA model (the correct model). Modest decrements in fit for the GFI and Mc indexes were observed, with .01 to .02 coefficient differences observed. None of the other indexes evidenced any average discrepancy.

DISCUSSION

The results of the study address the concern that GC model parameter estimates become biased when manifest variable autocorrelations are present. Table 5 summarizes the original findings and Table 6 shows the follow-up findings. These results suggest that researchers using GC models should consider as alternative competing models GCs that specify, a priori, AR, MA, or ARMA processes as well, because not doing so may result in inaccurate results. Given the widely reported finding that errors tend to correlate in longitudinal data, this would seem to be an important research issue to address.

It appears that when a GC model is fit to data possessing an AR process alone the fit of the model turns out to be poor, even though a viable GC process is present in the data. Concern in this case is less about the possibility that a researcher may incorrectly interpret the biased parameter estimates, given that the researcher is likely to not interpret the model results at all given its poor fit values. It may happen that the researcher instead moves to a different set of models altogether or simply

TABLE 5
Summary of Findings

	GCAR (.25)	*GCMA (.20)*	*GCARMA (.25/20)*
Intercept mean	Upward biased	Unbiased	Unbiased
Slope mean	Upward biased	Unbiased	Unbiased
Intercept variance	Upward biased	Upward biased	Upward biased
Slope variance	Upward biased	Unbiased	Unbiased
Slope-intercept covariance	Upward biased	Downward biased	Downward biased
Average fit index results	GFI = .50	GFI > .95	GFI > .95
	CFI = .81	CFI > .95	CFI > .95
	SRMR = .30	SRMR < .05	SRMR < .05

Note. This table summarizes the effect of unmodeled ARMA processes on GC parameter estimates. GCAR = growth curve autoregressive; GCMA = growth curve moving average; GCARMA = growth curve autoregressive moving average; GFI = Goodness of Fit Index; CFI = Comparative Fit Index; SRMR = Standardized Root Mean square Residual Estimate.

TABLE 6
Summary of Follow-Up Findings

	GCAR (AR = .15, .05)	*GCMA (MA = .1, .3, .5)*	*GCARMA (AR & MA=.1,.3,.5)*
Intercept mean	Upward biased	Unbiased	Unbiased
Slope mean	Upward biased	Unbiased	Unbiased
Intercept variance	Upward biased	Upward biased	Upward biased
Slope variance	Upward biased	Unbiased	Unbiased
Slope-intercept covariance	Upward biased	Downward biased	Downward biased
Average fit index results	When AR ≤ .05 GFI > .95 CFI > .95 SRMR < .05	GFI > .95 CFI > .95 SRMR < .05	GFI > .95 CFI > .95 SRMR < .05

Note. This table summarizes the effect of unmodeled ARMA processes on GC parameter estimates

quits. So, this study would advise that when a researcher has at least four occasions and a GC model does not fit well, it is reasonable to consider specifying an AR component in the model to rule this possibility out. Given the prevalence of AR (simplex) models in longitudinal research, it is reasonable to test the GCAR as a rival hypothesis.

This study would also caution researchers from fitting a GC model without considering as a rival hypothesis a GCMA or GCARMA model. GC models, it has been observed, often fit data with an MA or ARMA process very well. Given that the slope–intercept relation may be underestimated due at least in part to the increase in the intercept variance, it is reasonable to consider testing this possibility in practice. MA models and ARMA models are two of the three stationary models that Box and Jenkins (1976) identified as commonly found in time series data. Both MA and ARMA processes along with AR processes form a tight family of stationary models. Given that AR (simplex) processes have been quite often found in longitudinal data as well, it would be logically consistent to test for the presence of the MA and ARMA counterparts.

The utility of the results reported in this article are delimited by the conditions considered. The purpose of this study was to investigate what effect unmodeled ARMA processes have on GC parameters. Now that such evidence exists, future studies should focus on other relevant conditions that apply to longitudinal researchers. Although the findings of this study indeed suggest that unmodeled ARMA processes will effect GC parameters, it is unknown how many occasions are needed to identify such an effect. This study considered eight occasions. A study focusing on fewer occasions is necessary to determine whether the conclusions of the study hold.

Although this study does not consider squared autocorrelation processes higher than .25 for the AR and .25 for the MA, it is reasonable to conclude that if certain GC parameters are affected by .25 AR and .25 MA processes, they would also be affected by parameters exceeding these coefficients, respectively. What is not known, perhaps, is whether other GC parameters, now viewed as unaffected by AR or MA processes, will indeed be affected if the coefficient for the process were increased high enough. This point made, it is also true that mathematical limits exist for the correlation values of AR and MA processes. The standardized equations require mathematically that as AR or MA coefficients increase, the magnitude of other model parameters or the error term must decrease. At some limit, all other parameters staying the same, as either the AR or MA coefficients increase, the error in the model must decrease to zero. Likewise, at some limit, holding the error in the model constant, as either the AR or MA coefficients increase, the growth parameters must decrease. The premise of this study was that researchers most interested in the findings would have a strong theoretical reason for hypothesizing growth, but perhaps would have to contend with correlated errors. Giving the growth parameters' sufficient prominence in the data and allowing for some uncertainty (error), it was decided that no more than moderate AR and MA parameter conditions would be mathematically possible for this study. It is important to note that the condition in this study of a squared .25 AR or MA coefficient is actually a .50 AR or MA coefficient. (The .50 MA coefficient was considered in the follow-up studies reported in the results.) Therefore, enough room was allowed in the simulated variable variances for prominent AR and MA effects.

Yuan, Marshall, and Bentler (2003) treated the issue of model misspecification on parameter estimates in more general terms. Yuan et al., although advocating reasoned approaches to model modification, cautioned that even after modeling the effects of latent variables, manifest variables may still be correlated due to, among other things, test–retest effects characteristic of longitudinal data. Yuan et al. found that estimated parameters in a misspecified model may not be biased much by the misspecification so long as these parameters are not closely related to the misspecification. The results of this study suggest that certain GC parameters are closely related to AR parameters, whereas others are more closely related to MA parameters as the degree of bias for given GC parameters depends on which process is at hand. Interpretation of the GC parameters without recognizing and modeling AR or MA processes will lead to a misunderstanding of the GC results. New procedures for identifying the best models in the face of misspecification are becoming more prominent. Such procedures offer promise for identifying not only the ARMA processes discussed in this article, but other time-varying processes that could very well bias parameter estimates. For example, Marcoulides and Drezner (2001), and more recently Marcoulides and Schumacker (2004), indicated that many researchers acknowledge that SEM model search procedures are going to become an unavoidable step in applied SEM research. This conclusion is consistent with the finding of Yuan et al. (2003) that misspecifications differentially affect parameters according to their connection with such parameters in the model. Without search routines how is one to know where such misspecifications may exist and consequently which estimated parameters are otherwise biased? These search routines are computationally intensive in terms of the number of equations that must be considered, but there has been some recent success in the way of new algorithms capable of automating SEM specification searches (Marcoulides & Schumacker, 2004). These procedures will be particularly helpful in identifying time-dependent processes such as ARMA processes in the face of random sampling fluctuations.

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